B.Sc. Part I (Hons.) 1st Paper

Trigonometry (contd.)

HYPERBOLIC FUNCTIONS

Important formulae

$$\cosh x = \frac{1}{2}(e^x + e^{-x})$$

$$\cos ix = \cosh x$$

$$\sin ix = i \sinh x$$

$$\sinh x = \frac{1}{2}(e^x - e^{-x})$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$\cosh^2 x + \sinh^2 x = \cosh 2x = 2\cosh^2 x - 1 = 1 + 2\sinh^2 x$$

$$\sinh 2x = 2 \sinh x \cosh x$$

Q. Separate $\cot(\alpha+i\beta)$ into real and imaginary parts.

Soln.

$$\cot(\alpha+i\beta) = \frac{\cos(\alpha+i\beta)}{\sin(\alpha+i\beta)} = \frac{2\cos(\alpha+i\beta)}{2\sin(\alpha+i\beta)}$$

$$= \frac{2\cos(\alpha+i\beta)}{2\sin(\alpha+i\beta)} \times \frac{\sin(\alpha-i\beta)}{\sin(\alpha-i\beta)}$$

$$\Rightarrow \cot(\alpha+i\beta) = \frac{2\cos(\alpha+i\beta)\sin(\alpha-i\beta)}{2\sin(\alpha+i\beta)\sin(\alpha-i\beta)}$$

$$\Rightarrow \cot(\alpha+i\beta) = \frac{\sin[(\alpha+i\beta)+(\alpha-i\beta)] - \sin[(\alpha+i\beta)-(\alpha-i\beta)]}{2[\sin^2\alpha - \sin^2 i\beta]}$$

$$= \frac{\sin 2\alpha - \sin 2i\beta}{\cos 2i\beta - \cos 2\alpha}$$

$$\Rightarrow \cot(\alpha+i\beta) = \frac{\sin 2\alpha - i\sinh 2\beta}{\cosh 2\beta - \cos 2\alpha} \quad \left[\because \sin i\theta = i\sinh\theta,\ \cos i\theta = \cosh\theta\right]$$

$$\Rightarrow \cot(\alpha+i\beta) = \frac{\sin 2\alpha}{\cosh 2\beta - \cos 2\alpha} - i\frac{\sinh 2\beta}{\cosh 2\beta - \cos 2\alpha}.$$

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